

Задание 1: Типовые

$$(a) \lim_{x \rightarrow \infty} \frac{3x^4 - x^2 + 4}{5x^4 + 4x^2 + 1} = \left(\frac{\infty}{\infty} \right) = \lim_{x \rightarrow \infty} \frac{3 - \frac{x^2}{x^4} + \frac{4}{x^4}}{5 + \frac{4x^2}{x^4} + \frac{1}{x^4}} = \frac{3}{5}$$

$$(b) \lim_{x \rightarrow 2} \frac{x^2 + 6x - 16}{3x^2 - 5x - 2} \quad \text{Однот.} : \lim_{x \rightarrow \infty} \frac{3x^4 - x^2 + 4}{5x^4 + 4x^2 + 1} = \frac{3}{5}$$

$(x^2 + 6x - 16)$ (корни -8 и 2 по т. Виета) или

$$D_1 = 3^2 - 4 \cdot (-16) = 9 + 16 = 25 \quad \sqrt{D_1} = \sqrt{25} = 5$$

$$x_1 = \frac{-3 - 5}{2} = -8$$

$$x_2 = \frac{-3 + 5}{2} = 2$$

$$\text{или } (3x^2 - 5x - 2)$$

$$D = (-5)^2 - 4 \cdot 3 \cdot (-2) = 25 + 24 = 49 \quad \sqrt{D} = \sqrt{49} = 7$$

$$x_1 = \frac{5 + 7}{2 \cdot 3} = \frac{12}{6} = 2$$

$$x_2 = \frac{5 - 7}{2 \cdot 3} = -\frac{2}{6} = -\frac{1}{3}$$

$$\lim_{x \rightarrow 2} \frac{(x+8)(x-2)}{3(x-2)(x+\frac{1}{3})} = \frac{(x+8)}{3x+1} = \frac{2+8}{6+1} = \frac{10}{7} \text{ - Ответ}$$

$$\text{Однот.} : \lim_{x \rightarrow 2} \frac{x^2 + 6x - 16}{3x^2 - 5x - 2} = \frac{10}{7}$$

$$(b) \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{5x} \right)$$

$$(1 - \cos x) = \frac{x^2}{2}$$

$$\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{5x} \right) = \frac{x^2}{10x} = \frac{x}{10} = 0$$

$$\text{Ответ} : \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{5x} \right) = 0$$

$$(2) \lim_{x \rightarrow \infty} \left(\frac{x^2 + 1}{x^2} \right)^{x^2 + 1}$$